

A model finding a new Richardson potential with different scales for confinement and asymptotic freedom, by fitting the properties of Δ^{++} and Ω^- .

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Abstract

Phenomenological Richardson potential has built in asymptotic freedom (AF in short) and confinement, with only one parameter Λ in the potential. But it is known that the scales of AF and confinement are not the same. In the present work a relativistic mean field calculation for baryons is tried out with two parameters Λ and Λ' for AF and confinement respectively .

To test the two parameter potential we calculate the energies and the magnetic moments, of the triple u - quark system (Δ^{++}) and the triple s - quark system (Ω^-) and found good values for $\Lambda = 100 \text{ MeV}$ and $\Lambda' = 350 \text{ MeV}$. So we believe that the modified Richardson potential should have AF scale $\Lambda = 100 \text{ MeV}$ and the confinement scale $\Lambda' = 350 \text{ MeV}$.

Keywords: Hartree-Fock –Richardson Potential– magnetic moment – dense matter.

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1 Introduction

't Hooft suggested that the inverse of the number of colors N_c could be used as an expansion parameter in the otherwise parameter free QCD theory [1]. By the end of 90's, properties of large N_c baryons have been extensively studied by algebraic methods for spin and isospin symmetry. It has become possible to make a unified view on the various effective theories such as the Skyrme model, the non-relativistic quark model and the chiral bag model [2]. Witten [3] suggested that for large N_c baryons a mean field description could be obtained using a phenomenological interquark potential tested in the meson sector¹

Indeed self consistent baryon mass calculation is feasible with success in the mean field level [4] using the Richardson potential [5] as an interquark interaction. Richardson potential takes care of the two features of the qq force, asymptotic freedom and confinement as given below :

$$V(r_{12}) = - \frac{N_c^2 - 1}{2N_c} \frac{6\pi}{33 - 2N_f} [\Lambda^2 r_{12} - f(\Lambda r_{12})] \quad (1)$$

where $-\frac{N_c^2 - 1}{2N_c}$ is the color contribution, N_f is the number of flavors, taken to be three, and

$$f(t) = 1 - 4 \int_1^\infty \frac{dq}{q} \frac{\exp(-qt)}{[\ln(q^2 - 1)]^2 + \pi^2} \quad (2)$$

Λ is a parameter whose value was originally chosen as 400 MeV as for small q^2 the potential reduces to a linear confinement, and the linear confinement string tension from lattice calculation is about that value. However, the asymptotic freedom part also has the same Λ . In other words, both the confinement and asymptotic freedom scales are chosen to be the same in the potential equation (1) even though it is known that the asymptotic freedom scale should be around 100 MeV.

Same potential has been included in the relativistic HF calculation of strange quark matter to form compact stars. These stars are more compact than the conventional neutron stars and fit into the Bodmer-Witten hypothesis for the existence of strange quark matter. They lead to systems more compact than those derived assuming the bag model type of quark confinement [6, 7, 8] and leads to various observable phenomenon [9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. But Λ parameter had to be within 100 – 200 MeV. This is consistent with asymptotic freedom scale but for confinement scale, it is low.

Since the scales of these two phenomena, need not be the same as in the original potential., we incorporate them explicitly as Λ' and Λ :

$$V(r_{12}) = - \frac{N_c^2 - 1}{2N_c} \frac{6\pi}{33 - 2N_f} [\Lambda'^2 r_{12} - f(\Lambda r_{12})] \quad (3)$$

¹We agree with the referee that a relativistic Faddeev calculation would be more appropriate for a 3 quark system but do not attempt that. This is because our goal is to find an *effective interaction* from finite system for *use* in strange star calculations.

Λ' corresponds to the confinement part and Λ corresponds to the asymptotic freedom part.

Recently the magnetic moment of Δ^{++} was derived from experimental data [19] and it was shown in [20] that many models of hadrons cannot fit its value satisfactorily. We re-visit the relativistic mean field calculation [4] with two parameter Richardson potential (3). Elsewhere the star calculation is repeated with the modified potential ([21]).

In this work an extensive search is performed to find the magnetic moment and mass of the Δ^{++} and Ω^- . It is found that for the confinement part a Λ' of about 340-350 MeV and for asymptotic part a Λ of 100 MeV reproduces both the magnetic moment and the energy satisfactorily.

This relatively small value of asymptotic freedom parameter (Λ) is in agreement with Shifman *et al* [22] (70 MeV – 100 MeV).

Interestingly, in a recent paper, Radzhabov and Volkov [23] proposed that Λ' should be 340 MeV.

2 Details of calculation

Assuming the quarks occupy the same orbital, the mean field $\omega_{av}(r)$ is given by

$$\omega_{av}(\vec{r}) = -\frac{N_c^2 - 1}{2N_c} \int \phi_{jm}^\dagger(\vec{r}') V(\vec{r} - \vec{r}') \phi_{jm}(\vec{r}') d\vec{r}' \quad (4)$$

where $-\frac{N_c^2 - 1}{2N_c}$ is the color contribution in the color singlet state, the Fock term. The color factor for the Hartree term is zero. The energy E_{HF} is given by

$$E_{HF} = N_c \left(\epsilon - \frac{1}{2} \langle \phi_{jm} | \omega_{av} | \phi_{jm} \rangle \right) \quad (5)$$

where ϵ is the single particle energy obtained by solving

$$[\vec{\alpha} \cdot \vec{p} + \beta m + \omega_{av}(r)] \phi_{jm}(r) = \epsilon \phi_{jm}(r) \quad (6)$$

where α 's and β are the usual Dirac matrices. For quarks in the lowest ($1s_{1/2}$) orbital, one may write

$$\phi_{1s_{1/2}}(r) = \left[\frac{1}{4\pi} \right]^{\frac{1}{2}} \begin{pmatrix} iG(\vec{r})\chi_m \\ \vec{\sigma} \cdot \hat{r} F(\vec{r})\chi_m \end{pmatrix} \quad (7)$$

where χ_m is the Pauli spinor and the eqn(5) yields the system of coupled differential equations:

$$\frac{dG}{dr} - (m - \omega_{av} + \epsilon)F = 0 \quad (8)$$

$$\frac{dF}{dr} + \left(\frac{2}{r} \right) F + (\epsilon - \omega_{av} - m)G = 0 \quad (9)$$

The above equations are solved self-consistently by iteration since ω_{av} depends on Dirac large and small components $G(r)$ and $F(r)$.

In eqns (8) and (9), the self consistent single particle confining potential is a vector one, leading to instability in the solution. The same problem was encountered by Crater and Van Alstine [24] who suggested a prescription of taking a half-vector half-scalar form for the linear part. This choice also leads to a cancellation of spin-orbit effects at long range. So we add the confining part of the two-body potential equally to the energy and mass (1/2 vector, 1/2 scalar potential).

The vector and scalar potentials are respectively:

$$V_{vec}(r_{12}) = -\frac{N_c^2 - 1}{2N_c} \frac{6\pi}{33 - 2N_f} \left[\frac{\Lambda'^2 r_{12}}{2} - f(\Lambda r_{12}) \right] \quad (10)$$

$$V_{scalar}(r_{12}) = -\frac{N_c^2 - 1}{2N_c} \frac{6\pi}{33 - 2N_f} \frac{\Lambda'^2 r_{12}}{2} \quad (11)$$

The vector potential $V_{vec}(r_{12})$ is used in the expression of $\omega_{av}(\vec{r})$ [equation 4] and the scalar potential $V_{scalar}(r_{12})$ is added with the mass terms in the coupled differential equations.

The magnetic moment μ is given by (see [25]):-

$$\mu = -e_B \frac{2}{3} \int_0^\infty G(r)F(r)r^3 dr \quad (12)$$

where e_B is the charge of the baryon.

It is convenient to express the Dirac components $G(r)$ and $F(r)$ as sum of oscillators :

$$G(r) = \sum_n C_n R_{n0} \quad (13)$$

$$F(r) = \sum_m D_m R_{m1} \quad (14)$$

where C 's and D 's are coefficients. The expansion reduces the differential equation to an eigenvalue problem. The solution is to be found self consistently by diagonalizing the matrix and putting back the coefficients till convergence is reached. In general $R_{nl}(r)$ is given by :

$$R_{nl}(r) = \sqrt{\frac{2n!}{\Gamma(n+l+\frac{3}{2})}} r^l \exp(-\frac{1}{2}r^2) L_n^{l+\frac{1}{2}}(r^2) \quad (15)$$

where $L_n^{l+\frac{1}{2}}(r^2)$ are Associate Laguerre polynomials.

In the calculation, r is replaced by r/b in the expression of $L_n^{l+\frac{1}{2}}(r^2)$ where b is the oscillator length which need not to be the same for $G(r)$ and $F(r)$ and they are denoted by b and b' respectively.

Starting with a trial set of C's and D's , ω_{av} is calculated, a matrix is constructed and diagonalized. The new coefficients C_n and D_n can be read off from the eigenvector with the corresponding eigenvalue ϵ . The wave functions are put back in the equations and the process is continued till self consistency is reached.

But the problem is that the RHF solutions violate translational invariance, since they are formed by single-particle wave functions derived by an average potential ω_{av} . As a consequence, the centre-of-mass momentum is not well-defined in RHF solutions and this entails a spurious contribution from the centre-of-mass [CM] kinetic energy to the total energy. Since the relative importance of this effect increases as the number of particles decreases, it is important that it should be corrected for systems formed of few particles. This can be done by extending to the RHF equations the Peierls-Yoccoz procedure of nuclear physics. The spurious contribution is denoted by T_{CM} and the baryon mass [M] has to be compared with the difference $E_{HF} - T_{CM}$. Here this spurious contribution has also been calculated and subtracted from E_{HF} to estimate the correct energy of the baryon.

We have also estimated the values of r.m.s. radius r_{av} and checked if the wave functions are normalized or not. The normalization factor is denoted by N.

r_{av} is given by:

$$r_{av} = \sqrt{\int_0^{r_{max}} (G(r)^2 + F(r)^2) r^4 dr} \quad (16)$$

N is given by:

$$N = \int_0^{r_{max}} (G(r)^2 + F(r)^2) r^2 dr \quad (17)$$

where r_{max} is the upper limit of integration and is taken as 5.0 fm to make $N \sim 1$.

3 Results for Δ^{++}

We start with the simplest system, the totally symmetric spin and isospin triple u-quark state. Here m_u is taken as 4 MeV. We checked the convergence in choosing the number of the oscillators. In figure [1] we see that the change in E_{HF} is 32 MeV when we increase the matrix dimension from 5 X 5 to 7 X 7, but only 16 MeV from 7 X 7 to 9 X 9. It is interesting to note that the magnetic moment remains almost same when we increase the dimension from 7 X 7 to 9 X 9. In table 1, it is shown that E_{HF} is almost independent of oscillator parameter b and b' and the mass after CM correction is 1171 MeV.

Now, we vary Λ and Λ' independently. We give details of Λ and Λ' variational results in table 2 and table 3 for matrix dimension 7 X 7 as an example. We find that E_{HF} varies widely with Λ' as expected since Λ' is the confining parameter. With $\Lambda' = 350 \text{ MeV}$ and $\Lambda = 100 \text{ MeV}$ (table [2]) Δ mass = 1250 MeV and its magnetic moment 5.77 magneton. Our final result is given in table [4] for 9 X 9 matrix. We get the Δ mass to be 1224 MeV and its magnetic moment $\mu = 6.15$ magneton.

Table 1: Variation of Hartree Fock energy, centre of mass kinetic energy, mass and magnetic moment of Δ^{++} with oscillator parameter b where $\Lambda = \Lambda' = 350 \text{ MeV}$; for 9 X 9 matrix.

b'	b	$E_{HF}(\text{MeV})$	$T_{CM}(\text{MeV})$	$M(\text{MeV})$	$\frac{\mu}{\mu_0}$	$r_{av}(\text{fm})$	N
0.60	0.78	1310	139	1171	6.75	1.30	0.988152
0.60	0.80	1307	138	1169	6.69	1.31	0.990868
0.60	0.82	1303	138	1165	6.56	1.32	0.995152
0.60	0.84	1301	130	1171	6.42	1.31	0.998549
0.60	0.86	1299	130	1169	6.30	1.31	1.000855
0.60	0.88	1298	127	1171	6.23	1.30	1.002296

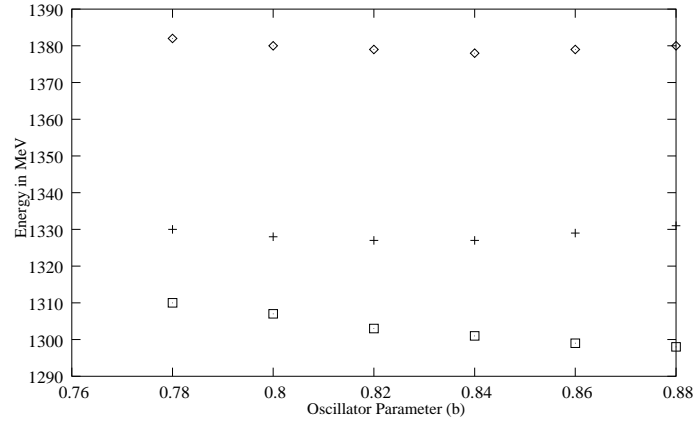


Figure 1: Variation of Hartree-Fock energy E_{HF} with Oscillator Parameter b for 5 by 5 , 7 by 7 and 9 by 9 matrices . Here \diamond stands for 5 by 5 matrices, $+$ stands for 7 by 7 matrices, \square stands for 9 by 9 matrices .

Table 2: Variation of Hartree Fock energy, centre of mass kinetic energy, mass and magnetic moment of Δ^{++} with asymptotic parameter Λ where Λ' is 350 MeV; b' is 0.60, b is 0.84 using seven by seven matrix.

$\Lambda(\text{MeV})$	$E_{HF}(\text{MeV})$	$T_{CM}(\text{MeV})$	$M(\text{MeV})$	$\frac{\mu}{\mu_0}$	$r_{av}(\text{fm})$	N
100	1391	141	1250	5.77	1.11	1.000001
250	1340	141	1199	5.95	1.13	1.000001
300	1333	140	1193	5.99	1.14	1.000000
325	1330	140	1190	6.00	1.14	1.000000
375	1325	140	1185	6.02	1.14	1.000000

Table 3: Variation of Hartree Fock energy, centre of mass kinetic energy, mass and magnetic moment of Δ^{++} with confinement parameter Λ' where Λ is 350 MeV; b' is 0.60, b is 0.84 using seven by seven matrix.

Λ' (MeV)	E_{HF} (MeV)	T_{CM} (MeV)	M(MeV)	$\frac{\mu}{\mu_0}$	r_{av} (fm)	N
250	1003	140	863	6.03	1.39	0.999999
300	1171	139	1032	6.09	1.22	1.000001
325	1251	140	1111	6.03	1.17	1.000001
375	1399	140	1259	6.16	1.14	1.000000
400	1475	142	1333	6.76	1.26	1.000000

Table 4: Hartree Fock energy, centre of mass kinetic energy, mass and magnetic moment of Δ^{++} where Λ' is 350 MeV, Λ is 100 MeV; b' is 0.60, b is 0.84 using nine by nine matrix.

E_{HF} (MeV)	T_{CM} (MeV)	M(MeV)	$\frac{\mu}{\mu_0}$	r_{av} (fm)	N
1354	130	1224	6.15	1.22	0.998476

Figures (2) and (3) show the wave functions and the single particle potential for $\Lambda' = 350 \text{ MeV}$ but $\Lambda = 350 \text{ MeV}$ and 100 MeV . As expected, there is no change in large r but an insignificant variation in small r . This is reflected in E_{HF} and μ (table 2).

On the other hand, Λ' variation (figures 4 and 5) is quite significant and larger in large r . This is reflected in E_{HF} and μ (table 3).

So we can conclude that for finite system calculations, the value of the confinement parameter Λ' plays a more important role in the Richardson Potential.

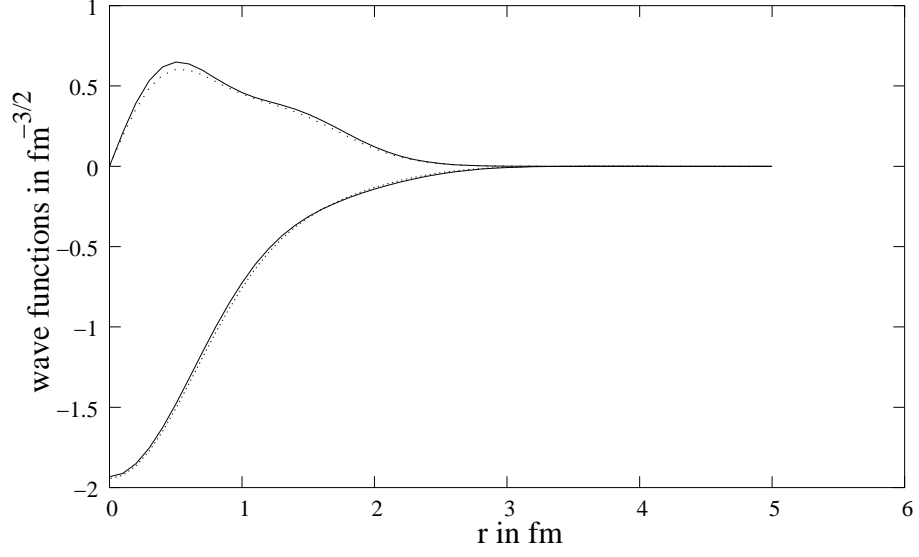


Figure 2: Variation of the wave functions with r for two different values of the asymptotic freedom parameter Λ (using seven by seven matrices); the upper ones are $G(r)$ and the lower ones are $F(r)$. Here the dotted curve is for $\Lambda = \Lambda' = 350 \text{ MeV}$ and the solid one is for $\Lambda = 100, \Lambda' = 350 \text{ MeV}$.

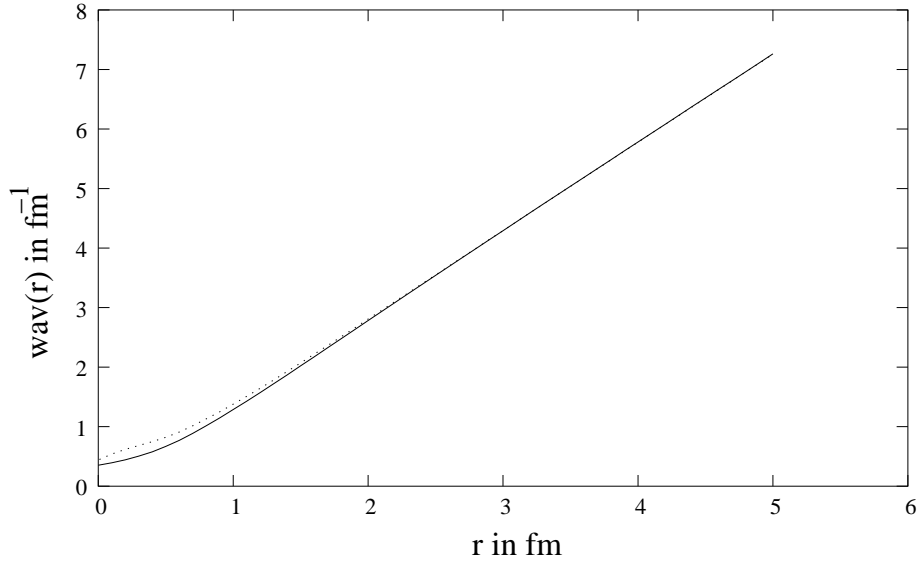


Figure 3: Variation of the single particle potential with r for two different values of the asymptotic freedom parameter Λ (using seven by seven matrices). Here the dotted curve is for $\Lambda = \Lambda' = 350 \text{ MeV}$ and the solid one is for $\Lambda = 100, \Lambda' = 350 \text{ MeV}$.

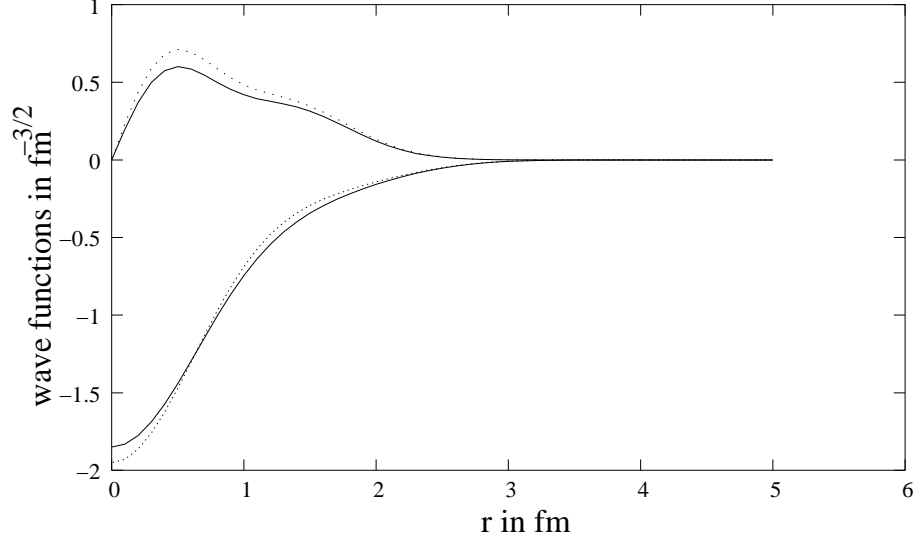


Figure 4: Variation of the wave functions with r for two different values of the confinement parameter Λ' (using seven by seven matrices); the upper ones are $G(r)$ and the lower ones are $F(r)$. Here the dotted curve is for $\Lambda' = 375 \text{ MeV}$, $\Lambda = 350 \text{ MeV}$ and the solid one is for $\Lambda' = 325 \text{ MeV}$, $\Lambda = 350 \text{ MeV}$.

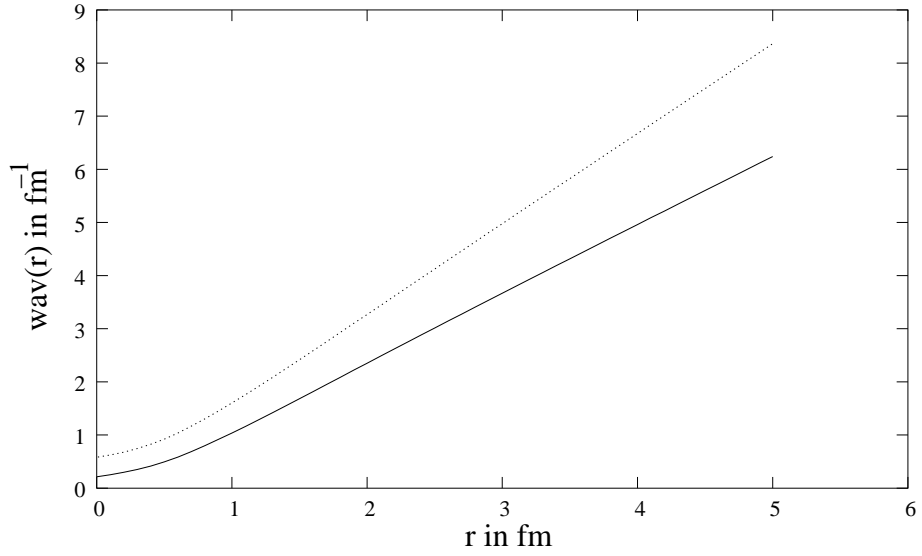


Figure 5: Variation of the single particle potential with r for two different values of the confinement parameter Λ' (using seven by seven matrices) . Here the dotted curve is for $\Lambda' = 375 \text{ MeV}$, $\Lambda = 350 \text{ MeV}$ and the solid one is for $\Lambda' = 325 \text{ MeV}$, $\Lambda = 350 \text{ MeV}$.

4 Result with Ω^-

Ω^- is the triple s-quark state. There is no qualitative change in the procedure except for the quark mass; m_s is taken here as 150 MeV. Results for $\Lambda' = \Lambda = 350$ MeV is shown in table [5].

Table 5: Variation of Hartree Fock energy, centre of mass kinetic energy, mass and magnetic moment of Ω^- with oscillator parameter b where $\Lambda = \Lambda' = 350$ MeV, b' is 0.60.

b	$E_{HF}(\text{MeV})$	$T_{CM}(\text{MeV})$	$M(\text{MeV})$	$\frac{\mu}{\mu_0}$	$r_{av}(\text{fm})$	N
0.60	1639	176	1463	-1.95	0.836879	1.000001
0.62	1638	175	1463	-1.95	0.836778	1.000000
0.64	1638	174	1464	-1.95	0.836879	1.000000
0.66	1638	172	1466	-1.95	0.836854	1.000001
0.68	1638	171	1467	-1.95	0.836882	1.000000
0.70	1637	169	1468	-1.95	0.83643	0.999999
0.72	1639	167	1472	-1.95	0.83695	1.00001

We improve the fit by taking different values for Λ' and Λ ; specifically for $\Lambda' = 350$ MeV and $\Lambda = 100$ MeV.

Table 6: Hartree Fock energy, centre of mass kinetic energy, mass and magnetic moment of Ω^- where Λ' is 350 MeV, Λ is 100 MeV; b' is 0.60, b is 0.70.

$E_{HF}(\text{MeV})$	$T_{CM}(\text{MeV})$	$M(\text{MeV})$	$\frac{\mu}{\mu_0}$	$r_{av}(\text{fm})$	N
1721	165	1556	-1.92	0.849015	0.999999

The centre of mass kinetic energy T_{cm} is rather high resulting a lower mass of Ω^-

5 Conclusions and summary

In summary we have shown that a RHF calculation can be done for the energies and magnetic moments of the simplest baryons Ω^- and Δ^{++} . We employed the oscillator basis with good convergence. Optimized oscillator parameters are used for the large and small components of the RHF wave functions. We believe that this model may be good base for further investigations of baryon properties. The center of mass correction is around 150 MeV for these baryons (about 10% of E_{HF}). Quark masses chosen are 4 and 150 MeV for u and s quark respectively. For q q interaction we chose Richardson potential. We separated out the confinement and asymptotic freedom scale parameter (Λ' and Λ). From the best fit of the energies of Ω^- and Δ^{++} we find $\Lambda' = 350 \text{ MeV}$ and $\Lambda = 100 \text{ MeV}$. We believe these are realistic values. Above parameters are now used in strange quark matter calculation ([21]).

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